King Fahd University of Petroleum and Minerals

College of Computer Science and Engineering
Information and Computer Science Department

ICS 253: Discrete Structures I
Summer Semester 2017-2018
Final Exam, Tuesday August 14, 2018.

Name:

ID#:

Instructions:

1. This exam consists of **nine** pages, including this page and the final reference sheet, containing **six** questions.
2. You have to answer all **six** questions.
3. The exam is closed book and closed notes. Non-programmable calculators are allowed. Make sure you turn off your mobile phone and keep it in your pocket if you have one.
4. The questions are **not equally weighed**.
5. This exam is out of **100** points.
6. You have exactly **120** minutes to finish the exam.
7. Make sure your answers are **readable**.
8. If there is no space on the front of the page, feel free to use the back of the page. Make sure you indicate this in order for me not to miss grading it.

Question Number	Maximum # of Points	Earned Points
I	**20**	
II	**15**	
III	**15**	
IV	**20**	
V	**15**	
VI	**15**	
Total	**100**	

I. (20 points) Choose the most correct answer from the following choices.

1. Let p = "I have a valid password" and q = "I log on to the server". The statement "It is necessary that I have a valid password in order for me to log on to the server." is expressed as
   (a) $p \rightarrow q$
   (b) $q \rightarrow p$
   (c) $\neg p \rightarrow q$
   (d) $p \leftrightarrow q$
   (e) $p \oplus q$

2. Freedonia has fifty senators. Each senator is either honest or corrupt. Suppose you know that at least one of the Freedonian senators is honest and that, given any two Freedonian senators, at least one is corrupt. Based on these facts, we can say that the number of Freedonian senators that are honest
   (a) is equal to 25 and the number of corrupt senators is equal to 25.
   (b) is equal to 1 and the number of corrupt senators is equal to 49.
   (c) is equal to 49 and the number of corrupt senators is equal to 1.
   (d) is equal to 2 and the number of corrupt senators is equal to 48.
   (e) cannot be determined for sure based on the given information.

3. Assuming that the universe of discourse (domain) for the variables x, y and z is the same, $\neg\forall x\big(\exists y\forall z P(x,y,z) \wedge \exists z\forall y P(x,y,z)\big)$ is equivalent to
   (a) $\exists x\big(\forall y\exists z\neg P(x,y,z) \wedge \forall z\exists y\neg P(x,y,z)\big)$
   (b) $\exists x\forall z\exists y\big(\neg P(x,z,y) \vee \neg P(x,y,z)\big)$
   (c) $\exists x\big(\forall z\exists y\neg P(x,z,y) \vee \forall z\exists y\neg P(x,y,z)\big)$
   (d) both (b) and (c).
   (e) none of the above.

4. Consider the following set of premises: "If I take the day off, it either rains or snows." "I took Tuesday off or I took Thursday off." "It was sunny on Tuesday." "It did not snow on Thursday." The following conclusion(s) can be drawn:
   (a) I did not took Tuesday off.
   (b) I took Thursday off and it snowed on Thursday.
   (c) I took Thursday off and it rained on Thursday.
   (d) Both (a) and (b)
   (e) Both (a) and (c)

5. Let A, B and C be sets. Then,
   (a) $(A - B) - C \subseteq B - A$
   (b) $(A - B) - C \subseteq B - C$
   (c) $(A - B) - C \subseteq C - A$
   (d) $(A - B) - C \subseteq A - C$
   (e) none of the above is true.

6. The recurrence relation $a_n = 8a_{n-1} - 16a_{n-2}$ has the following sequence(s) $\{a_n\}$ as solution(s):
   (a) $a_n = 1$
   (b) $a_n = 2^n$
   (c) $a_n = n4^n$
   (d) $a_n = n^2 4^n$
   (e) Both (c) and (d).

7. $\bigcap_{i=1}^{\infty}\left[\frac{1}{i}, 1+\frac{1}{i}\right] =$
   - (a) [0,1)
   - (b) (0,1]
   - (c) Φ
   - (d) {1}.
   - (e) none of the above.

8. Indicate the incorrect statement from the statements below
   - (a) The set of irrational numbers $\mathbb{R} - \mathbb{Q}$ is uncountable.
   - (b) The set $Z \times Z$ is countable.
   - (c) A subset of a finite set is finite.
   - (d) A subset of a countable set is countable.
   - (e) A subset of an uncountable set is uncountable.

9. The inequality $2^n > n^2$
   - (a) is true for all nonnegative integers n.
   - (b) is true for all $n \in Z^+$.
   - (c) is true for all $n \geq 4$.
   - (d) is true for all $n \geq 5$.
   - (e) is false for an infinite number of values $n \in Z^+$.

10. The following is/are linear homogeneous recurrence relation(s) with constant coefficients.
    - (a) $a_n = 3a_{n-2}$
    - (b) $a_n = a_{n-1} + 3$
    - (c) $a_n = a_{n-1} + 2a_{n-2} + na_{n-3}$
    - (d) $a_n = 3$
    - (e) (a), (b) and (d)

II. (15 points) In all questions below, make sure that you clearly justify your answer.

1. (5 points) What is the minimum number of students, each of whom comes from one of the 22 different Arab countries, who must be enrolled in a university to guarantee that there are at least 100 who come from the same country?

   100 = ceiling(N/22)

   N = 2179

2. (10 points) An arm wrestler is the champion for a period of 75 hours. (Here, by an hour, we mean a period starting from an exact hour, such as 1 p.m., until the next hour.) The arm wrestler had at least one match an hour, but no more than 125 total matches. Show that there is a period of consecutive hours during which the arm wrestler had exactly 23 matches.

III. (15 points) In all questions below, make sure that you clearly justify your answer.

1. (10 points) The English alphabet contains 21 consonants and five vowels. How many strings of six lowercase letters of the English alphabet contain
   (a) exactly two vowels?

   $21^4 * 5^2 * C(6,2)$

   (b) at least one vowel?

   $26^6 - 21^6$

2. (5 points) Find the value of

$$\sum_{j=1}^{100} \binom{100}{j} (3)^{100-j}$$

$(1+3)^{100} - 3^{100}$

IV. (20 points) In all questions below, make sure that you clearly justify your answer.

1. (5 points) Suppose that 50 people enter a contest and that different winners are selected at random for first, second, and third prizes. What is the probability that you are going to win one of these prizes if you are one of the contestants?

   3*49*48/50*49*48 = 3/50

2. (5 points) What is the probability that a five-card poker hand contains a straight, that is, five cards that have consecutive kinds? (Note that an ace can be considered either the lowest card of an A-2-3-4-5 straight or the highest card of a 10-J-Q-K-A straight. Also, note that the straight need not come from the same suit [otherwise it is called a straight flush]).

   $(10*C(4,1)^5)/C(52,5)$

3. (5 points) In a superlottery, a player selects 7 numbers out of the first 80 positive integers. What is the probability that a person wins the grand prize by picking 7 numbers that are among the 11 numbers selected at random by a computer.

   C(11,7)/C(80,7)

4. (5 points) Which is more likely: rolling a total of 10 when two dice are rolled or rolling a total of 10 when three dice are rolled? Clearly justify your answer.

   tedious

V. (15 points) In all questions below, make sure that you clearly justify your answer.

1. (9 points) Find the probability that a randomly generated bit string of length 10 begins with a 1 or ends with a 00 if the generated bits are independent and if the probability that a bit is a 1 is 0.6.

$0.6+0.4^4-(0.6*0.4*0.4)$

2. (6 points) Suppose that E and F are events such that $p(E) = 0.8$ and $p(F) = 0.6$. Show that $p(E \cup F) \geq 0.8$ and $p(E \cap F) \geq 0.4$.

VI. (15 points) In all questions below, make sure that you clearly justify your answer.

1. (7 points) A string that contains only 0s, 1s, and 2s is called a ternary string.
   (a) Find a recurrence relation for the number of ternary strings of length n that do not contain two consecutive 0s.

   (b) What are the initial conditions?

2. (8 points) Solve the following recurrence relation with together with the initial conditions given:

$$a_n = 7a_{n-1} - 10a_{n-2} \text{ for } n \geq 2, a_0 = 2, a_1 = 1.$$

Some Useful Formulas

$\mathbb{N} = \{0,1,2,3,\dots\}$ $\mathbb{Z} = \{\dots,-2,-1,0,1,2,\dots\}$ $\mathbb{Q}$ = set of rational numbers
$\mathbb{R}$ = set of real numbers

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \quad , \quad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \quad , \quad \sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$$

$$\sum_{i=0}^{n} a^i = \frac{a^{n+1}-1}{a-1} \text{ where } a \neq 1 \quad , \quad \sum_{i=0}^{\infty} a^i = \frac{1}{1-a} \text{ where } |a|<1,$$

$$\sum_{i=0}^{n} ic^i = \sum_{i=1}^{n} ic^i = \frac{nc^{n+2} - nc^{n+1} - c^{n+1} + c}{(c-1)^2}$$

$$\sum_{i=1}^{\infty} ia^{i-1} = \frac{1}{(1-a)^2} \text{ where } |a|<1$$

$p \rightarrow (p \vee q)$	Addition	$[\neg q \wedge (p \rightarrow q)] \rightarrow \neg p$	Modus Tollens
$(p \wedge q) \rightarrow p$	Simplification	$[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$	Hypothetical syllogism
$((p) \wedge (q)) \rightarrow (p \wedge q)$	Conjunction	$[(p \vee q) \wedge \neg p] \rightarrow q$	Disjunctive syllogism
$[p \wedge (p \rightarrow q)] \rightarrow q$	Modus Ponens	$[(p \vee q) \wedge (\neg p \vee r)] \rightarrow (q \vee r)$	Resolution

Some Useful Sequences	
n^2	1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, …
n^3	1, 8, 27, 64, 125, 216, 343, 512, 729, 1000, 1331, …
n^4	1, 16, 81, 256, 625, 1296, 2401, 4096, 6561, 10000, 14641, …
2^n	2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048, …
3^n	3, 9, 27, 81, 243, 729, 2187, 6561, 19683, 59049, 177147, …
$n!$	1, 2, 6, 24, 120, 720, 5040, 40320, 362880, 3628800, 39916800
f_n Fibonacci	1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, …

$A \cap U = A$ $A \cup \Phi = A$	Identity Laws	$A \cup U = U$ $A \cap \Phi = \Phi$	Domination Laws
$A \cap A = A$ $A \cup A = A$	Idempotent Laws	$A \cup B = B \cup A$ $A \cap B = B \cap A$	Commutative Laws
$\overline{(\bar{A})} = A$	Complementation Law	$A \cup (B \cup C) = (A \cup B) \cup C$ $A \cap (B \cap C) = (A \cap B) \cap C$	Associative Laws
$\overline{A \cap B} = \bar{A} \cup \bar{B}$ $\overline{A \cup B} = \bar{A} \cap \bar{B}$	De Morgan's Laws	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	Distributive Laws
$A \cup (A \cap B) = A$ $A \cap (A \cup B) = A$	Absorption Laws	$A \cup \bar{A} = U$ $A \cap \bar{A} = \Phi$	Complement Laws